From Biot-Savart to Circular Fields: Student Coordination of Magnetic Field Representations in AR

Michele W. McColgan,[1] George E. Hassel,[1] Kamyar Pashayi,[1] and Jason W. Morphew[2]

[1]*Department of Physics & Astronomy, Siena University, 515 Loudon Road, Loudonville, NY, 12211*
[2]*School of Engineering Education, Purdue University, 701 West Stadium Avenue, West Lafayette, IN 47907*

Students often struggle to coordinate right-hand rules, vector equations, and spatial representations when reasoning about magnetic fields. This study uses the Mathematical Sensemaking (MSM) framework to examine how students coordinate mathematical, physical, symbolic, and embodied representations while interacting with augmented reality (AR) models of the magnetic field of a current-carrying wire and the Biot-Savart law. Interviews were conducted with undergraduate students using MARVLS AR models, including long-wire and Biot-Savart visualizations with interactive highlighting features. This paper presents two contrasting case studies from the physics-student interview cohort. Both students productively used right-hand-rule gestures, AR vectors, and symbolic representations to reason within individual models. However, Student 1 used the AR environment as an external testing space to coordinate the local Biot-Savart field direction with the global circular magnetic field model, while Student 2 reasoned productively within each model but did not fully stabilize this cross-model coordination. These cases suggest that AR-supported mathematical sensemaking depends not only on access to multiple representations, but on how students use embodied interaction and interactive visual features to coordinate physical meanings across representations.

I. INTRODUCTION

Students often struggle to develop robust conceptual and spatial models of magnetic fields generated by current-carrying wires and the vector relationships represented by the Biot-Savart law. [1–4] Prior work has shown that students frequently rely on procedural applications of right-hand rules without developing coherent connections between vector equations, magnetic field direction, and three-dimensional field structure. Difficulties are especially pronounced for cross-product reasoning, where representational and spatial features can significantly impact students' ability to coordinate vector relationships and right-hand-rule gestures. [5–7]

Research has also shown that students are often more successful using the "curling fingers" right-hand rule for determining the magnetic field around a long straight wire than using the cross-product form associated with vector equations such as the Biot-Savart law. These difficulties are linked to challenges with spatial reasoning, vector algebra, and coordinating multiple representations simultaneously. [7–9]

Augmented reality (AR) environments provide opportunities for students to manipulate and visualize three-dimensional vector relationships through embodied interaction. Prior work by the authors with the MARVLS AR models has shown that students use gesture, spatial manipulation, and physical interaction with the Merge© Cube [10] to connect two-dimensional representations, vector equations, and three-dimensional magnetic field structures. [11–19]

This study uses the Mathematical Sensemaking (MSM) framework developed by Gifford and Finkelstein to analyze how students coordinate mathematical and physical reasoning while interacting with AR models. The MSM framework characterizes reasoning according to the type of tool used, mathematical or physical, and the type of object being interpreted, mathematical or physical. This produces four modes: mathematical tools used to reason about mathematical objects (Msm-M), mathematical tools used to reason about physical objects (Msm-P), physical tools used to reason about mathematical objects (Psm-M), and physical tools used to reason about physical objects (Psm-P). [20–22] In this paper, we use these modes to analyze how students coordinate general cross-product equations, the Biot-Savart equation, right-hand-rule gestures, highlighted AR vectors, two-dimensional dots/x's notation, and three-dimensional AR magnetic field representations.

In this work, we examine how students coordinate mathematical, physical, symbolic, and embodied representations while reasoning about the magnetic field of a current-carrying wire and the Biot-Savart law. We focus on two contrasting case studies to show that both students could coordinate representations within individual models, but differed in whether they used the AR environment to test consistency across the long-wire and Biot-Savart models. We use the MSM framework to examine how students coordinated the representational resources that emerged during interaction with the AR models. This paper addresses two research questions:

RQ1: How do students coordinate mathematical, physical, symbolic, and embodied representations while reasoning with AR models of the magnetic field of a current-carrying wire and the Biot–Savart law?
RQ2: What differences emerge between students who coordinate representations within individual models and students who coordinate across the long-wire and Biot–Savart models?

II. METHODOLOGY

This paper reports an exploratory comparative case analysis drawn from a larger interview group. The larger study included 27 semi-structured interviews: 16 with students who had completed or were enrolled in introductory electricity and magnetism and 11 with students without university-level physics instruction. Because the present paper focuses on coordination of the long-wire and Biot–Savart representations after instruction, only the physics-student interviews were considered for this analysis. Two physics students were selected as contrasting information-rich cases.

Students interacted with augmented reality (AR) visualizations within the MARVLS App using a Merge©Cube and smartphone interface while responding to think-aloud prompts concerning magnetic field direction, right-hand rules, vector relationships, and the Biot-Savart law. Interviews included over-the-shoulder screen recordings including audio and observational notes concerning gesture use and interaction with the AR models.

Two students from the physics cohort were selected for detailed case-study analysis because they demonstrated contrasting patterns of cross-representational coordination. Student 1 coordinated the long-wire and Biot-Savart models by using the AR environment, gestures, and highlighted vectors to test consistency across representations. Student 2 reasoned productively within both the long-wire and Biot-Savart models, but did not fully coordinate the local Biot-Savart field direction with the global circular magnetic field representation. Analysis proceeded in four stages. First, the interview recordings were segmented into reasoning episodes. An episode was defined as a continuous stretch of talk, gesture, and/or AR interaction in which a student used one or more representations to reason about a specific topic, such as the direction of the magnetic field at a point, the meaning of the 2D dots/x's notation, or the vector ordering in the Biot–Savart equation. Episode boundaries were noted when the interview prompt changed, when the student shifted to a new representation, or when the target of reasoning changed. Second, each episode was annotated for the representations used, including equation variables, AR vectors, highlighting buttons, RHR gestures, 2D dots/x's notation, cube rotation, and verbal explanations. Third, episodes were coded using the MSM framework by identifying the tool used by the student and the object being interpreted. For example, using highlighted AR vectors to identify ($\mathrm{d}\vec{l}$, $\hat{r}$, and $\mathrm{d}\vec{B}$) in the equation was coded as Psm-M, while using the Biot–Savart equation and RHR to

determine the local field direction was coded as Msm-P/Psm-P. Finally, episodes were compared within and across the two cases to identify whether coordination remained within a single model or connected the local Biot–Savart representation with the global circular field representation.

Through this analysis, we identified the representational resources students used while reasoning about the long-wire and Biot–Savart models. These included mathematical tools, mathematical objects, physical/embodied tools, and physical objects or meanings. Figure 1 summarizes these resources and the MSM links observed in the interviews. The figure also corresponds to features of the AR models and user interface, including equation-linked highlighting buttons, the long-wire highlighting button that connected 2D dots/x's notation to corresponding AR vectors, AR vector visualizations, and right-hand-rule gestures. Figure 1 provides a general framework for these possible links in the AR tasks, and the results compare how two students used these resources differently while reasoning about the magnetic field of a current-carrying wire and the Biot–Savart law.

Initial episode boundaries and MSM codes were proposed by the first author and then discussed with the research team using selected video excerpts, transcripts, and gesture notes. Disagreements were resolved through discussion by returning to the video record and clarifying the tool-object relationship in the episode. Because this paper presents exploratory case analysis, the goal was analytic transparency rather than frequency-based reliability claims.

Through the episode analysis, we identified the representational resources students used while reasoning with the long-wire and Biot–Savart models. Figure 1 summarizes these resources and the MSM links observed in the interviews. The figure also corresponds to features of the AR models and user interface: the long-wire model included a 3D circular field visualization, a 2D dots/x's representation, and a highlighting button linking the 2D symbols to AR vectors; the Biot–Savart model included equation-linked highlighting buttons, ($d\vec{l}$, $\hat{r}$, $d\vec{B}$) vectors, and a local point P.

III. RESULTS

Using the representational structure summarized in Fig. 1, we compare how two students coordinated mathematical, physical, symbolic, and embodied resources while reasoning about the long-wire and Biot–Savart AR models. These cases are not intended to classify students as successful or unsuccessful. Both students demonstrated productive coordination within individual representations. The contrast concerns a specific coordination challenge: whether the local $d\vec{B}$ direction from the Biot–Savart model was connected to the tangent direction of the global circular magnetic field around a long straight wire.

Student 1 demonstrated strong cross-model coordination. In the long-wire model, Student 1 connected current direction, the global RHR gesture, 2D dots/x's notation, the highlighting button, and 3D AR magnetic field arrows. While viewing the 2D representation, Student 1 explained that the magnetic field was "going out of the 2D representation" on one side and "going into the 2D representation" on the other. When using the Biot–Savart model, Student 1 coordinated the general cross-product structure with the specific vector ordering in the equation. After examining the AR vectors and equation-linked highlighting, Student 1 revised his RHR gesture until the vectors aligned, stating, "My first vector is $d\vec{l}$, my middle finger should point in $\hat{r}$, now it matches perfectly." Student 1 then connected this local field direction to the global circular field, explaining that the current in the wire created a magnetic field "curling in the same direction as this law." Gesture observations showed that Student 1 rotated his curled fingers so that the fingertips matched the local B-field direction at different positions around the wire. This rotational curled-finger gesture functioned as an embodied bridge between the local straight B-field vector and the tangent direction of the global circular field. Figure 2 illustrates Student 1 using both the Biot–Savart cross-product RHR and the global magnetic-field RHR while viewing the AR models.

Student 2 also made several productive representational connections. In the long-wire model, Student 2 used the global RHR gesture to connect current direction with circular magnetic field direction, explaining, "Current is going up and then your fingers wrap, that is the direction of the B field." Student 2 also interpreted the 2D dots/x's representation and corrected the interpretation after using the highlighting feature, stating that "the ones with the circles go out of the page and the x's go in." In the Biot–Savart model, Student 2 used the equation-linked highlighting buttons while naming the vectors in the AR model and connected the Biot–Savart RHR to local B-field direction. Student 2 identified the local B-field direction as down on one side of the wire and up on the other. Thus, Student 2's difficulty was not a general inability to use the Biot–Savart representation. Rather, the difficulty appeared when Student 2 attempted to coordinate the local Biot–Savart result with the global circular field representation. Although Student 2 used both RHR systems, Student 2 did not fully stabilize that the thumb represents $d\vec{B}$ in the Biot–Savart RHR while the curled fingers represent $\vec{B}$ in the global long-wire RHR. When asked why the two B-field directions should match, Student 2 stated, "It makes sense but I don't know how to explain it."

The episodes in Table I show that successful AR-supported mathematical sensemaking involved more than the presence of multiple representations. Both students used RHR gestures, interpreted AR vectors, and connected symbolic representations to physical field directions. The difference was whether these resources were used to coordinate physical meanings across models. Student 1 treated the AR environment as an external testing space for comparing the Biot–Savart and long-wire representations. Student 2 used the same representational resources productively within individual models, but the relationship between the local straight B-field vector and the tangent direction of the global circular field remained less stable.

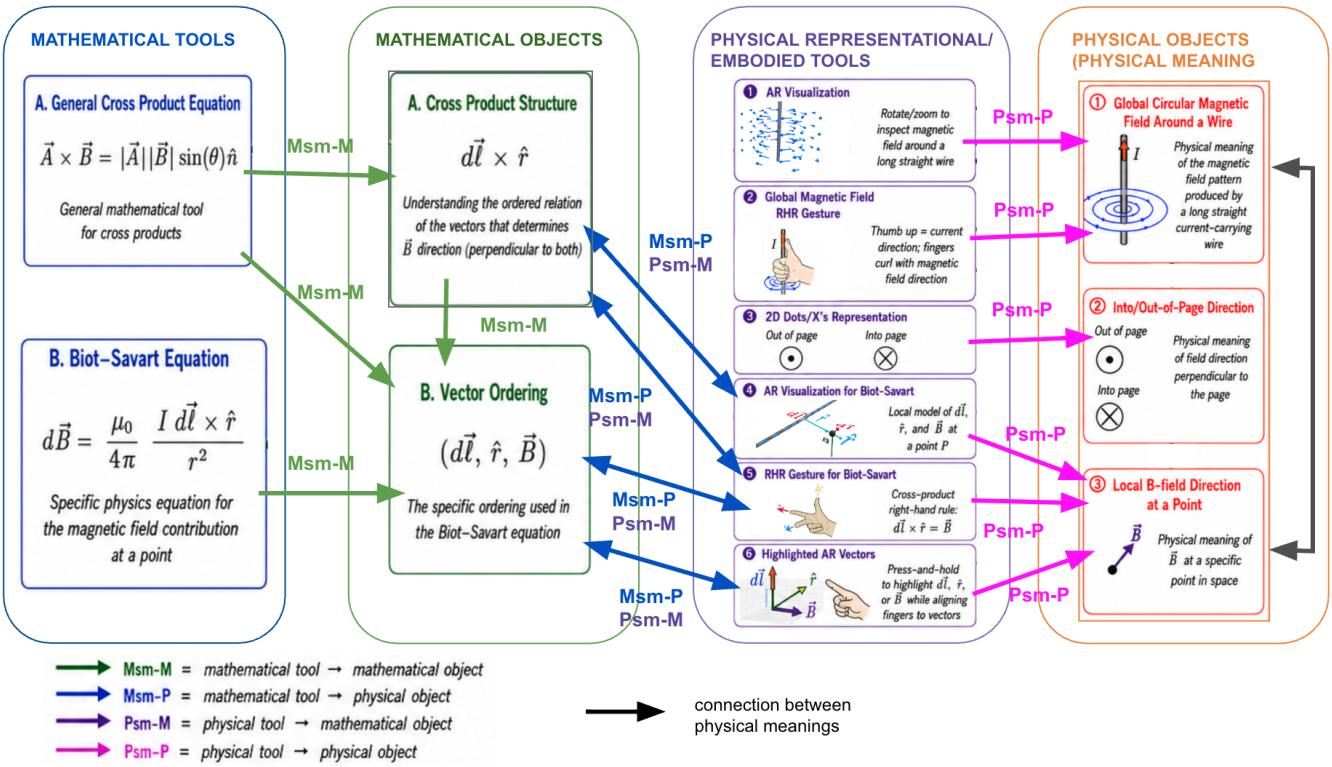


FIG. 1. General MSM Framework for Coordinated Mathematical and Physical Representations for AR-supported reasoning about the magnetic field of a current-carrying wire and the Biot-Savart law.

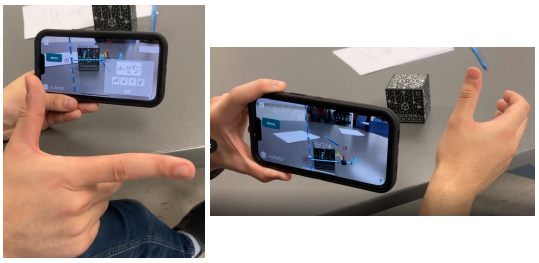


FIG. 2. Student 1 coordinating right-hand-rule gestures with AR representations. (a) Student 1 uses the cross-product RHR while interacting with the Biot–Savart AR model. (b) Student 1 uses the global magnetic-field RHR while interacting with the long-wire AR model. In both cases, the gesture is positioned near the phone display, supporting comparison between hand orientation and the AR vector representation.

IV. DISCUSSION & CONCLUSION

The comparison of these two cases suggests that AR-supported mathematical sensemaking involves more than providing multiple representations. Both students demonstrated productive coordination within individual models, so the cases should not be read as successful versus unsuccessful. Rather, the contrast concerns one specific coordination challenge: whether the local Biot–Savart $d\vec{B}$ direction was connected to the tangent direction of the global circular field. Student 1 used the AR environment as an external testing space, aligning gestures, highlighted vectors, and field representations across the long-wire and Biot–Savart models. Student 2 reasoned productively within each model, but did not fully coordinate the equivalence between the local and global B-field representations. These findings also highlight the importance of representational design in AR learning environments. The long-wire model included a 3D field visualization, a global RHR gesture affordance, a 2D dots/x's representation, and an interactive highlighting feature connecting the 2D symbols to AR vectors. The Biot-Savart model emphasized vector ordering, highlighted AR vectors, and the cross-product equation. Future versions of the models could make these representational systems more parallel by adding an equation representation to the global long-wire model and a 2D symbolic representation to the Biot-Savart model. These additions would not guarantee cross-model coordination, but they could provide more explicit opportunities for students to compare local and global magnetic field representations.

The authors acknowledge support from their National Science Foundation award 2120446.

TABLE I. Representative episodes of representational coordination across MSM modes

Student	Representational Link	Evidence	MSM mode(s)	Interpretation
S1	Coordinates long-wire AR model, global RHR, 2D dots/x's representation, and highlighting button	"going out of the 2D representation" and "going into the 2D representation"; uses the button to highlight dots/x's and corresponding AR vectors	Psm-P	Uses gesture, AR vectors, 2D notation, and interactive highlighting to connect the global circular field with local into/ out-of-page field directions
S1	Uses highlighted AR vectors and local Biot-Savart RHR gesture to interpret Biot-Savart vector ordering	"My first vector is $d\vec{l}$, my middle finger should point in $\hat{r}$, now it matches perfectly"	Psm-M	Uses physical/ embodied tools, including highlighting and gesture, to make sense of the mathematical object of cross-product vector ordering.
S1	Connects Biot-Savart equation and local RHR to local $d\vec{B}$ direction	"If I have my first finger pointing in the direction of $d\vec{l}$, and my middle finger pointing in $\hat{r}$, then my thumb points exactly where the magnetic field vector is"	Msm-P/ Psm-P	Coordinates the Biot-Savart equation, RHR gesture, and local physical field direction at a point
S1	Coordinates local Biot-Savart field direction with global circular magnetic field	"The current in the wire is creating a magnetic field that's curling in the same direction as this law"	Psm-P	Uses AR and rotational curled-finger gesture to connect the local straight B-field vector with the tangent direction of the global circular field
S2	Coordinates long-wire AR model, global RHR, 2D dots/x's representation, and highlighting button	"Current is going up and then your fingers wrap" and "the circles go out of the page and the x's go in"; uses the button to highlight dots/x's and corresponding AR vectors	Psm-P	Correctly uses the global RHR, 2D notation, and interactive highlighting to interpret circular field direction and into/out-of-page field direction
S2	Uses Biot-Savart equation buttons/ highlighting to identify vectors in the AR model	Uses buttons while naming $\hat{r}$, $d\vec{l}$, and $\vec{B}$	Psm-M/ Msm-P	Uses highlighted AR vectors to interpret equation variables and uses equation variables to interpret physical vectors in the model
S2	Connects Biot-Savart local RHR to local $\vec{B}$-field direction	B-field "down" on one side and "up" on the other	Msm-P/ Psm-P	Productively coordinates the Biot-Savart cross product, local RHR gesture, and local field direction
S2	Attempts to coordinate local $d\vec{B}$ direction with global circular field	"It makes sense but I don't know how to explain it"	Incomplete Psm-P	Does not fully stabilize that the thumb represents $d\vec{B}$ in the Biot-Savart local RHR while curled fingers represent $\vec{B}$ in the global RHR; the straight local $d\vec{B}$ vector is not fully connected to the tangent of the circular field

[1] McColgan, M., Finn, R., Broder, D., Hassel, G. *Electricity and Magnetism Conceptual Assessment*, Phys. Rev. Phys. Ed. Res., **13**. (2017).

[2] Li, J. and Singh, C. *Developing and Validating a Conceptual Survey to Assess Introductory Physics Students' Understanding of Magnetism*, Eur. J. Phys. **38**, (2017).

[3] Singh, C. *Improving Student Understanding of Magnetism*,arXiv:1701.01523.

[4] Chasteen S., Pollock, S. J., Pepper, R. E., Perkins, K. K., *Thinking like a physicist: A multi-semester case study of junior-level electricity and magnetism*, AJP **80**, 923, (2012).

[5] Knight, *The vector knowledge of beginning physics students*, TPT, **33**, 74 (1995).

[6] Chabay, R. and Sherwood, B., *Restructuring the introductory electricity and magnetism course*, AJP **74**, 329, (2006).

[7] Kustusch, M. B., *Assessing the impact of representational and contextual problem features on student use of right-hand rules*, Phys. Rev. Phys. Educ. Res., **12**, 1 (2016).

[8] Donhauser, A., Küchemann, S., Kuhn, J., Rau, M., Malone, S., Edelsbrunner, P., and Lichtenberger, A., *Making the invisible visible: Visualization of the connection between magnetic field, electric current, and Lorentz force with the help of augmented reality*, TPT, **58**, 6 (2022).

[9] Deprez, T., Gijsen, S. E., Deprez, J., and De Cock, M., *Investigating student understanding of cross products in a mathematical and two electromagnetism contexts*, Phys. Rev. Phys. Educ. Res. **15**, 2 (2019).

[10] Merge Cube, *AR/VR Learning and Creation*, https://mergeedu.com/cube.

[11] McColgan, M., MARVLS AR for Physics 2 [Mobile App] Version 1.2.2, 2025. App Store. https://apps.apple.com/us/app/marvls-ar-for-physics-2/id6630392503

[12] McColgan, M., MARVLS AR Physics 2 E&M [Mobile App]. Version 3.4.2, 2025. Google Play Store. https://play.google.com/store/apps/details?id=com.SienaARResearch.SienaEMApp

[13] McColgan, M., MARVLS Apps and Lessons, marvls.net

[14] McColgan, M. W., Hassel, G. E., Pashayi, K, and Morphew, J. W., *Seeing Electromagnetic Induction: Student Sense Making with Augmented Reality*, Proceedings of the 2026 ASEE Annual Conference, 2026.

[15] McColgan, M. W., Hassel, G. E., Pashayi, K, and Morphew, J. W., *MSM Framework: Augmented Reality Models of 3D Vectors*, Paper presented at 2025 ASEE Annual Conference and Exposition, Montreal, Quebec, Canada. doi:10.18260/1-2–56981

[16] McColgan, M. W., Hassel, G. E., and Pashayi, K., *MSM Framework: AR Model of the Force on a Charge Moving in a Magnetic Field*, 2024 PERC Proceedings [Boston, MA, July 10-11, 2024], edited by Q. X. Ryan, A. Pawl, and J. P. Zwolak, doi:10.1119/perc.2024.pr.McColgan.

[17] Bennett, J. A., Morphew, J. W. McColgan, M. W.,*Embodied Learning with Gesture Representation in an Immersive Technology Environment in STEM Education* Paper presented at 2024 ASEE Annual Conference and Exposition, Portland, Oregon.

[18] McColgan, M. W., Hassel, G. E., Stagnitti, N. C., Morphew, J. W., and Lindell, R. S., *Understanding Magnetism Concepts Through Augmented Reality: A Qualitative Analysis* Paper presented at 2024 ASEE Annual Conference and Exposition, Portland, Oregon.

[19] McColgan, M. W., Hassel, G. E., Stagnitti, N. C., Morphew, J. W., and Lindell, R. S., *Augmented Reality to Scaffold 2D Representations of 3D Models in Magnetism*, 2023 PERC Proceedings [Sacramento, CA, July 19-20, 2023], edited by D. L. Jones, Q. X. Ryan, and A. Pawl, doi:10.1119/perc.2023.pr.McColgan.

[20] Gifford J. D., & Finkelstein, N. D. *Categorizing Mathematical Sense Making and an Example of How Physics Understanding Can Support Mathematical Understanding*, 2020 PERC Proceedings [Virtual Conference, July 22-23, 2020], edited by S. Wolf, M. B. Bennett, and B. W. Frank, doi:10.1119/perc.2020.pr.Gifford.

[21] Gifford J. D., and Finkelstein, N. D. *Categorical framework for mathematical sense making in physics*, Phys. Rev. Phys. Educ. Res. **16**, (2020).

[22] Gifford J. D., and Finkelstein, N. D., *Applying a Mathematical Sense-Making Framework to Student Work and Its Potential for Curriculum Design*, Phys. Rev. Phys. Educ. Res. **17**, (2021).